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LETTER TO THE EDITOR

ASYMPTOTIC CONCENTRATION DISTRIBUTION OF AN INVOLATILE SOLUTE IN AN EVAPORATING DROP

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RECENTLY Schlünder [1] derived expressions giving the ratio with respect to time, of the surface concentration c_{r_0} to the mean concentration c_m of an involatile solute in an evaporating drop. He showed and confirmed experimentally that the ratio approached a constant asymptotic value which, under the circumstances that he noted, was a good approximation to the real ratio over much of the lifetime of the drop.

Schlünder found it impossible to obtain a closed form for the solution of his differential equations without making the assumptions that both the radius of the surface of the drop r_0 and (dr_0/dt), where t is time, were constant. In justification he pointed out that the product $r_0\,(dr_0/dt)$ is experimentally almost constant. The purpose of this communication is to point out that a closed form is obtainable for the asymptotic concentration distribution by making the single assumption that $r_0(dr_0/dt)$ is constant. The result is very simple.

The diffusion equation for spherical symmetry and without convection is:

$$D\left[\frac{\partial^2 c}{\partial r^2} + \frac{2}{r}\frac{\partial c}{\partial r}\right] = \frac{\partial c}{\partial t} \tag{1}$$

where D is diffusivity, c is concentration and r is the radial co-ordinate.

Changing to the moving co-ordinate system $R = (r/r_0)$, equation (1) becomes:

$$D\left[\frac{\partial^2 c}{\partial R^2} + \frac{2}{R}\frac{\partial c}{\partial R}\right] = r_0{}^2\frac{\partial c}{\partial t} - r_0 R\frac{\partial c}{\partial R}\frac{\partial r_0}{\partial t} \tag{2}$$

where the last term accounts for the movement with respect to time of the co-ordinate r where R remains constant.

The assumptions that $r_0(dr_0/dt)$ is constant is put in the form

$$r_0{}^2 = -Kt \tag{3}$$

where K is a constant and the time t is always negative. Equation (2) now becomes

$$\frac{\partial^2 c}{\partial R^2} + \frac{\partial c}{\partial R}\left[\frac{2}{R} - \frac{R}{2}\frac{K}{D}\right] = -\frac{K}{D}t\frac{\partial c}{\partial t} \tag{4}$$

The asymptotic state requires that, for a given value of R, $c = c'(-t)^{-3/2}$, where c' is a constant, since the concentration is inversely proportional to the volume of the drop. Equation (4) now becomes:

$$\frac{\partial^2 c}{\partial R^2} + \frac{\partial c}{\partial R}\left[\frac{2}{R} - \frac{R}{2}\frac{K}{D}\right] - \frac{3}{2}\frac{K}{D}c = 0 \tag{5}$$

The boundary condition is:

$$\frac{\partial c}{\partial R} = 0 \text{ when } R = 0$$

and the solution to equation (5) is:

$$c = c_0 \exp\left[\left(\frac{K}{4D}\right)R^2\right] \tag{6}$$

where c_0 is the concentration at the centre of the drop.

The properties of equation (6) describing the concentration distribution are so obvious that they require no

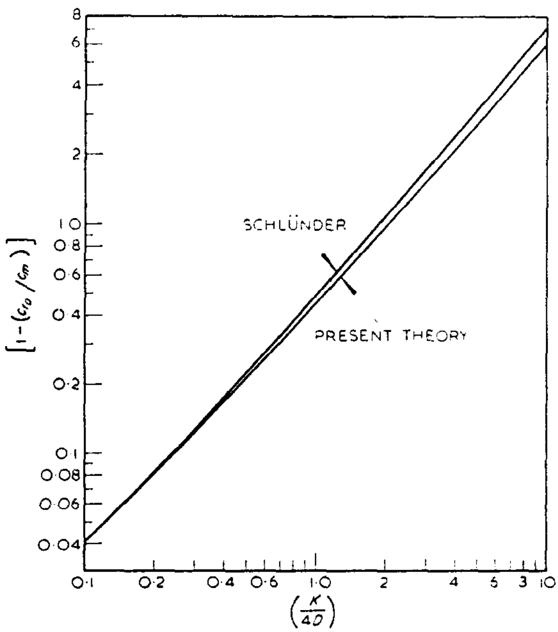

FIG. 1. Asymptotic ratio of surface to mean concentration.

discussion except to state that the simplicity of equation (6) makes the presentation of the exact theory worthwhile.

The mean concentration in the drop is obtained in the usual manner using equation (6) and is:

$$c_m = 3c_0 \int_0^1 R^2 \exp\left[\left(\frac{K}{4D}\right) R^2\right] \mathrm{d}R \tag{7}$$

and, directly from equation (6), the concentration at the surface of the drop is:

$$c_{r_0} = c_0 \exp\left(\frac{K}{4D}\right) \tag{8}$$

The exponential term in equation (7) can be put in series form to perform the integration.

The ratio (c_{r_0}/c_m) has been calculated and is plotted in Fig. 1. The asymptotic value given by Schlünder is also shown for comparison and is seen to vary little from the exact value.

When $(K/4D)$ is small, it can be estimated from equations (7) and (8) that

$$\frac{c_{r_0}}{c_m} \simeq 1 + a\left(\frac{K}{4D}\right) \tag{9}$$

where the exact theory gives $a = 0{\cdot}4$ and Schlünder estimates that $a = 0{\cdot}5$.

ACKNOWLEDGEMENT

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REFERENCES

1. E. U. Schlünder, Temperatur- und Masseänderung verdunstander Tropfen aus reinen Flüssigkeiten und wässrigen Salzlosungen, *Int. J. Heat Mass Transfer* 7, 49–73 (1964).

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